Mathematics Grades 6-8

NUMBER STRAND COMPANION DOCUMENT

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Description and Rationale

In Mathematics, learners require a balanced pedagogical approach to ensure that they can independently apply mathematical concepts in all contexts of their lives. Mathematics instruction cannot be only educator-directed, learners must have opportunities to experience mathematics through direct instruction, experiential learning, inquiry-based, and relevant learning.
Holding each learner in the highest regard and creating a safe and positive space in mathematics classrooms are both vital components when developing independent mathematics learners with confident and positive mindsets.

Professional learning and school(s)-based teaming are two components that strengthen an educator's mathematical pedagogy and content knowledge. Educators are encouraged to connect with their school administrator, school-based teams, district subject coordinators and district math leads/coaches for support related to your professional needs in mathematics education.

As Cummins and Early (2015) point out: "language is infused in all curricular content" (p.11). This means that all teachers are teachers of language. Instruction for **multilingual language learners (MLLs)** and first-language English speakers alike must include the development of academic language. High yield strategies for supporting learners with the language of *mathematics* are included throughout this document.

Contexts and Concepts- Grades 6, 7, and 8

Context	Application			
Concepts	**Strategies:** Determining when to use algorithms, mental procedures, technology/ tools, or other strategies Determining appropriate units of measure Discovering the most efficient strategies Determining the reasonableness of an answer and explaining thinking or rules Verifying solutions with substitution	**Processes:** Developing mathematical rules and algorithms Using manipulatives Modeling Determining unit rate Creating graphs	**Fluency:** Conversion between formats, representations, and equivalents of numbers Ways to present data Automaticity (with perfect squares) Scale	**Communication:** Representing mathematics concretely, pictorially, and symbolically Algebraic expressions Using formal mathematical vocabulary Using formal mathematical symbols Decimal & Bar notation; introducing terms repeating and period Equations

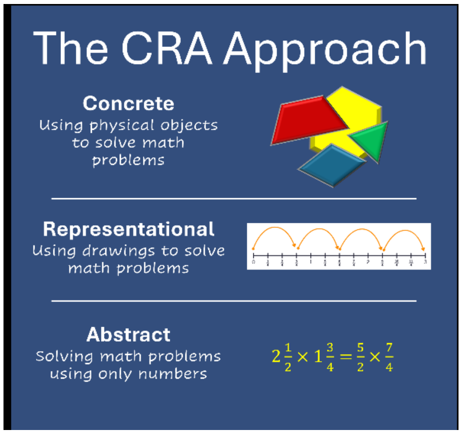
				Using variables and coefficients Using the congruence symbol

Strength-based practices develop a learner's ability to connect prior knowledge to new mathematical concepts. A personalized approach to teaching mathematics is developed and sustained by collaborative, school-based, math teams. Math *Professional Learning Communities*™ continue to be a major predictor in achievement success and positive attitudes of mathematics educators and learners. Davies, Herbst, and Parrott-Reynolds (2012) suggest three parts to beginning with the end in mind when planning for all learners (p. 31-32):

- Unpack all standards or learning outcomes.
- Work across grade levels to revisit syllabi, curriculum maps, and learning pathways.
- Analyze student work with colleagues.

Representation: Concrete, Pictorial, and Symbolic

A comprehensive grasp of mathematical concepts is attained when learners apply genuine understanding to solve problems, facilitated by opportunities to utilize tools for representing mathematical ideas. Learners indicate that their cognitive processes in mathematics engage five brain pathways, including two visual pathways. Thus, fostering a visual approach to mathematical thinking, beyond numerical understanding, is crucial.



Using concrete, pictorial, and symbolic models *(i.e., concrete, representational, abstract or C.R.A. model)* during direct instruction, learner-led practice, and interventions solidifies concepts and supports metacognitive development.

To foster fluency and flexibility in mathematics, learners require robust concrete and visual representations to grasp concepts symbolically (i.e., abstract). Utilizing math manipulatives, such as 3D objects for hands-on exploration, enhances learners' comprehension.

Below are some suggested concrete materials:

Algebra Tiles	Fraction Blocks	Pattern Blocks
Attribute Blocks	Fraction Circles	Pentominoes
Balance Scales (pan or beam)	Fraction Strips	Polydrons
Base-ten Blocks	Geoboards	Standard & Non-standard measuring tools
Coloured Tiles	Geometric Solids	Tangrams
Cuisenaire Rods®	Geo Strips	Trundle Wheel

Dice (number cubes and other versions)	Linking Cubes	Two Coloured Counters
Dominoes	Miras®	3-D Shapes, Nets & Skeletons

Below are some suggested pictorial(representational) materials:

Area Model	Flow Chart	Number Line (regular and open)
Arrays	Geometric Templates	Place Value Language Cards
Charts and Graphs (e.g., bar, pie, pictorial)	Graph Paper	Ratio Table
Carroll and Venn Diagrams	Graphic Organizers	Spinners
Coordinate Grids	Hundred Chart	Table of Values
Decimal Squares®	Hundredths Circle	Thousandths Grids
Double Number Line	Number Frames (e.g., 5, 10, 20, 100, 1000)	

Educators are reminded to review the Provincial Assessment Guide Criteria regarding manipulative use during provincial assessments.

Connection: Leveraging Prior Knowledge

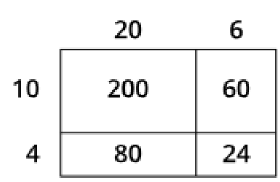
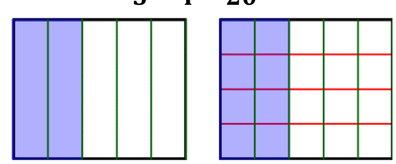
Understanding that early years' concepts are building blocks for middle level mathematics is key when educators plan lessons and interventions. Teaching learners *how* to make connections in mathematics improves conceptual mathematical understanding and increases independence. ***Metacognition development*** is an essential component for future-ready learner success. When learners habitually reflect on their personal problem-solving processes it fosters a deeper understanding of how to approach mathematical tasks.

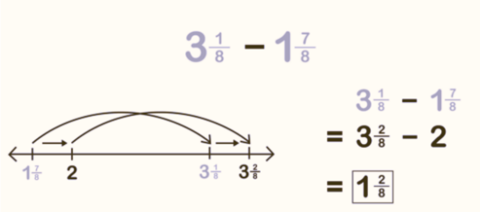
Below are some examples of questions that develop metacognitive skills:

- How does this question connect to what I already know?
- What strategies can I use to solve this problem?
- What do I need to pay more attention to when solving these types of problems?
- How can I explain my strategy to a classmate?
- Where have I seen these types of questions in real life?
- What follow up questions do I have?

Properties and Concepts

Drawing on math knowledge from early years benefits learners as they navigate more intricate and abstract skill descriptors in middle school. Presented below are various examples illustrating the concepts and properties introduced across K-8 mathematics that bridge both elementary and middle school.

Name	Description	Elementary Example	Middle Example
Associative property	A property of addition and multiplication states grouping numbers to be added or multiplied can be changed without affecting the operation's outcome. The associative property simplifies computation.	1 + 7 + 9 (7 + 9) + 1 = 7 + (9 + 1)	4 x 5 x 8 (8 × 4) × 5 = 8 × (4 × 5)
Area model	A diagram uses area to display mathematical concepts (i.e., most commonly multiplication). The length and width of the rectangle represent the factors, the area of the rectangle represents the product.	 26 × 14 = 200 + 60 + 80 + 24 = 364	$\frac{2}{5} \times \frac{4}{4} = \frac{8}{20}$ 
Benchmarking	Relating or comparing numbers to a benchmark number develops an understanding of number magnitude, master foundational facts, and improve number and operational sense.	371 + 198 200 would be the benchmark number replacing 198. It is a multiple of ten and easier to compute. 371 + 200 =571 The difference between 198 and 200 is 2. The number 2 is then taken away from the sum giving an answer of 569.	When comparing equivalent fractions and equivalent decimals, use a double number line to anchor learner's prior knowledge and facilitate math discussion. 
Commutative property	A property of **addition** and **multiplication** whereby changing the **order** does not	12 + 26 = 38 26 + 12 = 38 OR	8 x 17 = 136 17 x 8 = 136 OR

	affect the result. The commutative property can simplify computation.	a + b = b + a	a x b = b x a
Compensation	A strategy when part of the value of one number is given to another number to make computation easier.	26 + 99 thought of as 25 + 100 1 from the 26 is transferred to the 99 to make 100 26 + 99 thought of as 26 + 100 = 126 Since 1 too many was added, subtract 1 from 126 to answer of 125	1.6 + 0.99 thought of as 1.59 + 1.00 0.01 from the 1.6 is transferred to the 0.99 to make 1.00 1.6 + 0.99 thought of as 1.6 + 1.0 = 2.6 Since 0.01 too many was added, subtract 0.01 from 2.6 to answer of 2.59
Constant difference	The same quantity can be added to (or subtracted from) each number in a subtraction computation without affecting the answer (difference).	1001 – 398 Often regarded as a difficult subtraction because it involves regrouping across zeros. However, if 2 is added to each number in the computation, the answer (difference) will be the same and the computation is simpler. 1001 – 398 = *x* *BECOMES* 1003 – 400 = 603	
Decompose & Compose numbers	Decomposing (breaking apart) numbers allows learners to see the relationship between a whole number and its parts and develops fluency with all numerals. Composing numbers putting back together of numbers that have been decomposed.	Decompose example, the number 17 is usually taken apart as 10 + 7. It can also be taken apart as 8 + 9, or 8 + 8 + 1, and so forth. Compose example, to solve 24 + 27, a learner might decompose the numbers as 24 + 24 + 3, then recompose the numbers as 25 + 25 + 1 to give the answer 51.	Decompose example, the fraction $\frac{7}{8}$ can be taken apart as $\frac{5}{8}+\frac{2}{8}$ It can also be taken apart as $\frac{4}{8}+\frac{2}{8}+\frac{1}{8}$ Composing example, to solve 5.7 + 3.8, a learner might decompose the numbers as 5.5 + 3.5 + 0.5, then recompose the numbers as 6.0 + 3.5 to give the answer of 9.5
Estimation	Strategies used to obtain an approximate answer. Used when an exact answer is not required and when checking reasonableness.	*Clustering*: A strategy used for estimating the sum of numbers that cluster around one value. For example, the numbers 42, 47, 56, and 55. (cluster is around 50) So, the estimate is 50 + 50 + 50 = 200. *Compatible Numbers:* A strategy that involves using numbers that are easy to work with. For example, to estimate the sum of 28, 67, 48, and 56, one could round and add the multiples of ten 30 + 70 + 50 + 60.	

		Front End Estimation: A strategy that involves the addition of significant digits (highest place value), with an adjustment of the remaining values. For example: Step 1 – Add the left-most digit in each number. 193 + 428 + 243 Think 200 + 400 + 200 = 800. Step 2 – Adjust the estimate to reflect the size of the remaining digits. 93 + 28 + 53 Think 100 + 25 + 50 = 175. Think 700 + 175 = 875.	
Partner numbers	When numbers are combined to make groups of 10. Decomposing numbers as a skill readiness skill that will help learners be able to use partner numbers as a strategy consistently. Making tens is a helpful strategy in learning foundational facts for all numerals.	If a learner knows that 6 + 4 = 10, then the learner can surmise that 6 + 5 equals 1 more than 10, or 11. This is a useful strategy for adding a series of numbers (i.e., 4 + 5 + 6 + 2 + 8, find combinations of 10 first [4 + 6, 8 + 2] and then add any remaining numbers).	If a learner knows that 0.7 + 0.3 = 1.0, then the learner can surmise that 0.6 + 0.3 equals 0.1 less than 1.0, or 0.9 If a learner knows that $\frac{3}{8} + \frac{5}{8}. = 1$, then the learner can surmise that $\frac{4}{8} + \frac{5}{8} = \frac{9}{8}$ or $1\frac{1}{8}$

Collaboration: Mathematical Discourse

The mathematics classroom values learner experiences, understanding, and supports inquiry. Learners need time and space to explore problem-solving situations when developing personal strategies. Learners must realize that it is acceptable to solve problems in different ways and that solutions, at times, may vary.

Creating varied workspaces in classrooms to facilitate collaboration and communication is an essential component of learner success, (e.g., spaces for large and/or small group work, partner work, and independent work).

Mathematical discourse in a safe and positive space is a necessity. Discourse increases conceptual understanding, justification of skills, and opportunities for learners to refine their strategies.

Instructional Practices

High yield instructional math practices are essential in fostering effective learning environments and maximizing learner comprehension and retention. By emphasizing conceptual understanding, problem-solving skills, and real-world applications, these practices aim to cultivate a deep-rooted understanding of mathematical concepts.

- **Explicit Instruction:** Educators modeling a skill and verbalizing their thinking using clear and consistent language.
- **Visuals**: Educators using concrete, pictorial, and symbolic representations with all concepts. Learners should understand how to use these three types of models/representations to explore new concepts and to demonstrate their thinking. (e.g., base ten blocks, pattern blocks, number lines, graphic organizer (Frayer Model) etc.)

- **Spaced practice:** Learners are assessed on concepts consistently in regular intervals. This assessment technique improves the learner's ability to retrieve information and eventually store the information in their long-term memory. (e.g., Assessment of a concept on day one, then two days later, and then longer intervals. These could be low-stakes or no-stakes assessments, however once a concept is introduced it never really is "done".)

- **Retrieval practice:** A practice that helps learners retain information. Learners are given opportunities to connect prior knowledge to new challenges. This is not merely rote practice, retrieval practice includes higher order thinking questions, "think-pair-share" opportunities, and introduction of new contexts.

- **Interleaving:** Chunking or breaking up a concept into small pieces. These smaller pieces should be woven with smaller pieces of a different topic allowing the brain to make further concept connections. (e.g., Teaching the operations of addition and subtraction at the same time as they are reciprocal operations.)

CULTURALLY AND LINGUISTICALLY INCLUSIVE INSTRUCTION

<table>
<tr><th>Support Comprehensible Input</th><th>Support Comprehensible Output</th><th>Support Real Life Connections</th></tr>
<tr>
<td>Clarify instructions and other texts by simplifying, chunking, and adding visuals.

E.g. Before: Explore the classroom, looking for 2-D shapes. Record your observations with tally marks.

After:

Walk around the school.

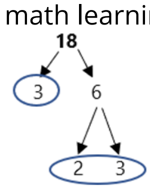
Look for 2-D shapes.

Record your observations with tally marks.</td>
<td>Incorporate structured conversations (provide a prompt and a sentence frame for responding. Model first, then have students exchange responses orally in small groups).

E.g. Prompt: Use a fraction to describe this picture.

Sentence frame: _____ of the animals in this picture are _____.

Model: $\frac{1}{3}$ of the animals in this picture are dogs.</td>
<td>Use visual, graphic, and sensory supports (Hyunh and Skelton, 2023).

Visual Supports: use photographs or illustrations to support math learning


Graphic Supports: use charts, graphs, tables, and timelines to support math learning
<table>
<tr><th colspan="2">Factor Pairs of 42</th></tr>
<tr><td>1</td><td>42</td></tr>
<tr><td>2</td><td>21</td></tr>
<tr><td>3</td><td>14</td></tr>
<tr><td>6</td><td>7</td></tr>
</table>

Sensory Supports: use videos, manipulatives, music, drama tools, sorting cards, models, or other multi-sensory tools to support math learning

</td>
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</table>

Concept Elaboration: Number Sense

Place Value and Magnitude

In middle school, learners extend their understanding of numbers up to one million and beyond, including billions, trillions, etc. Opportunities for learners to explore place value patterns happen when they have practice reading numbers in different ways, reading numbers greater than a million, recording numbers in **standard form (8 100 000)** and **decimal notation (8.1 million)** and when they can establish personal referents for larger numbers (e.g., the school theatre seats 600 people). Numbers extend to infinity to the left and into ten thousandths, hundred thousandths, and millionths places to the right.

The place value system follows patterns such as:

- each position represents ten times as much as the position to its' right
- each position represents one-tenth as much as the position to its' left
- positions are grouped in threes for reading numbers
- spaces (not commas) are used when writing numbers, except for 4-digit numbers (e.g., 5024; 3 506 980)

Understanding the relative size of numbers is a crucial part of developing number sense. Through experiences, learners develop flexibility in identifying and representing numbers beyond 1 000 000.

Instructional Strategies:

1. Ask learners to find various representations for multi-digit and decimal numbers. Encourage discussion on the need for accuracy in reporting these numbers and the appropriate use of rounded numbers.
2. Present a metre stick as a number line from zero to one billion. Ask learners where one million, half a billion, one hundred million, etc., would be on this number line.
3. Write decimals using place value language and expanded notation to help explain equivalence of decimals. **Notice below that "adding zeros" does not change has the magnitude.**

   0.2 = 2 tenths
   0.20 = 2 tenths + 0 hundredths
   0.200= 2 tenths + 0 hundredths + 0 thousandths
4. Ensure that proper vocabulary is used when reading all numbers. Provide opportunities for learners to read decimals in context. Saying decimals correctly will help make the connection between decimals and fractions (e.g., 5.0072 should be read as "five and seventy-two ten thousandths" not "five point zero, zero, seven, two").
5. Include contexts that lend themselves to using large numbers such as astronomical data and demographic data. Contexts that lend themselves to decimal thousandths include sports data such as race times and metric measurements.

Problem Solving with Large Numbers

It is important for middle school learners to practice solving problems with large numbers using all four operations (i.e., addition, subtraction, multiplication, and division). At times, using tools like calculators can be helpful, however learners should have experience calculating numbers of larger magnitude using their personal strategies. In such cases numbers must be friendly so that learners can connect to their prior knowledge. For example, when dividing a number like $12 000 000 among 3 people, using a personal strategy and connecting the problem to foundational division facts would be easier than a calculator. Always look at the situation and the numbers involved first, then decide the best way to problem solve.

Before or after doing a calculation, learners should always estimate to determine if the answer is reasonable. Just because a calculator gives an answer does not always mean it is correct.

Instructional Strategies:

1. Incorporate manipulatives and interactive activities to engage learners in hands-on learning experiences. Use tools like fraction circles, fraction bars, base-ten blocks, or interactive whiteboard apps to allow learners to manipulate and explore very small numbers.
2. Emphasize comparisons and equivalents to help learners understand the relationships between very small numbers and whole numbers. Encourage learners to compare fractions or decimals to benchmarks, such as 0, 0.5, or 1, to determine their relative sizes. Provide opportunities for learners to find equivalent fractions or decimals through various representations.
3. Engage learners in problem-solving tasks that require them to apply their understanding of very small numbers in different contexts. Present open-ended problems or scenarios that challenge learners to use fractions or decimals to solve practical problems. Encourage learners to explain their reasoning and justify their solutions.
4. Have learners research populations of cities and/or provinces in Canada and cities and/or countries of the world. Using this information, learners can estimate differences, compare populations, and draw conclusions about Canada compared to the world.
5. Include resources such as Statistics Canada, Canada Population Clock, World Population Clock, the Canadian Global Almanac, the Guinness Book of World Records, World Atlas, and the Top Ten of Everything. Use children literature such as: “If the World Were a Village” by David J. Smith, or “On Beyond a Million” by David Schwartz, to provide a context for large numbers to create and solve problems.

Factors and Multiples

Multiples of a whole number are the result of multiplying that number by any other whole number. For example, to find the first four multiples of 3, multiply 3 by 1, 2, 3, and 4 to get 3, 6, 9, 12. In the beginning, multiples can also be found by skip counting by that number, however learners should connect their prior foundational multiplication fact knowledge when exploring multiples.

Factors are numbers multiplied to get a product (e.g., 4 and 5 are factors of 20). Factors of a number can be found by dividing the number by smaller numbers and checking for a remainder of zero. Learners should quickly connect their prior foundational multiplication and division fact knowledge when exploring factors.

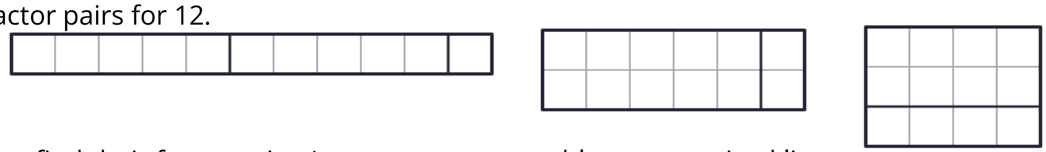
Key points about factors and multiples include:

- factors are never greater than the number itself (e.g., factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24)
- the greatest factor is always the number itself, and the least factor is one

- the second factor is always half the number or less (unless the number is prime) (e.g., multiples of 6: 6, 12, 18, 24…all have 6 as a factor)
- each whole number has a finite set of factors, but there is no limit to the multiples
- to reinforce understanding, students can explore these concepts and create their own definitions, such as "factor × factor = multiple".

Instructional Strategies:

1. Determine the factors of a number by arranging square tiles and/or using grid paper into as many different arrays (rectangles) as possible. Record the unit length and width of each rectangle. For example, if 12 tiles were used, the rectangles would be 1 by 12, 2 by 6, and 3 by 4. These are the factor pairs for 12.



2. Investigate numbers to find their factor pairs. Learners may use tables or organized lists to determine factors (i.e., begin with 1 and the number itself, then 2 or the next possible factor and its factor partner, etc.)

Factor Pairs of 42	
1	42
2	21
3	14
6	7

Prime and Composite Numbers

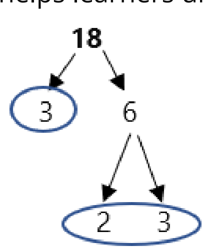
A **prime number** has only two factors: 1 and itself (e.g., 29 is prime because its only factors are 1 and 29). Prime numbers apply only to whole numbers. A **composite number** has more than two factors and includes all non-prime numbers other than one and zero (e.g., 9 has factors of 1, 3, and 9). Check out this lesson on Polypad on common factors using prime factor circles.

Neither 0 nor 1 are classified as prime or composite. One has only one factor (itself), and zero is not prime because it has an infinite number of divisors and is not composite because it cannot be written as a product of two factors that does not include 0.

Even numbers (other than 2) are readily identifiable as composites because they have a minimum of three factors: 1, 2, and the number itself.

Instructional Strategies:

1. Introduce the concept of prime factorization using a factor tree diagram. Start with a composite number and guide learners to break it down into its prime factors by repeatedly dividing it by prime numbers. This strategy helps learners understand how composite numbers are built from prime factors.



2. Use tables to represent and organize prime and composite numbers. For example, create a table with numbers listed vertically and mark which ones are prime, and which are composite. This visual representation helps learners grasp the concept more easily.
3. Engage learners with concrete models to explore prime and composite numbers. For example, provide models such as counters, blocks, or tiles and ask learners to create arrays to represent different numbers. Then, guide learners to identify which numbers have exactly two factors (prime) and which have more than two factors (composite).
4. Factor composite numbers that are odd (e.g., 33, 39). Learners sometimes mistake these for prime as they do not readily see how they are factored.
5. Explore the sieve of Eratosthenes to identify the prime numbers to 100. On a hundreds chart, have the learners begin by circling the first prime number, 2, and then cross out all of the multiples of 2 (composite numbers). Circle the next prime number, 3, and cross out all its multiples. Learners then proceed to the next number that has not been crossed off and repeat the procedure. At the end of the process the circled numerals will be the prime numbers to 100. Discuss any patterns they notice.

Fractions The Professional Learning Hub hosts professional learning for educators including modules on fractions.
In middle school, learners increase their fractional understanding by building on their prior knowledge of **factors** and **multiples**, **multiplication** and **division facts**, and the **concept of fair share. Educators in middle school note, fractions as a concept are introduced in grade 4 and addition and subtraction with fractions is introduced in grade 5.** The notion of "fairness" then extends to fractional parts. Learners use tools like number lines and other concrete models to visualize and comprehend fractions.

Models for Understanding

Learners must explore fractions with concrete and pictorial models to understand the five ways in which fractions can be represented. A comprehensive grasp of fractions is attained when learners apply genuine understanding to solve problems, facilitated by opportunities to utilize tools for representing fractions.

Using concrete, pictorial, and symbolic models *(i.e., concrete, representational, abstract or C.R.A. model)* during direct instruction, *learner*-led practice, and interventions solidifies understanding of fraction concepts. To foster fluency and flexibility with fractions, learners require robust concrete and visual representations to grasp fraction concepts symbolically (i.e., abstract).

To develop *fraction sense,* learners must understand that fractions are used in five different ways. As a result, educators need to realize that some models and representations may support one use of a fraction more precisely than another.

- **Part-whole**: The numerator of a fraction indicates the quantity of fractional parts being considered; the denominator specifies the type of fractional part being counted. Learners often struggle to grasp the concept of what constitutes the "whole." Concrete models and linear models are compatible for this use of fractions.

- **Fraction as a measure:** Describes a distance from zero. A linear model (line number) is the best model for learners. In this case the numerator is the measure from zero and the denominator is the total measure (length).
- **Fraction as a quotient:** When a fraction is used to represent division, this is simply a symbolic/abstract representation of a division expression (e.g., 16 ÷4 is the same as $\frac{16}{4}$ OR 1÷3 is the same as $\frac{1}{3}$)
- **Fraction as an operator:** A fraction has a function and either enlarges or reduces a quantity. Models and representations used here are the same as the ones used when solving problems that multiply with fractions.
- **Part-part:** The relationship between the numerator and denominator describes a proportional quantity. These fractions are often referred to as ratios. Two-coloured counters are a great way to model this use of fractions.

Decimals: Importance of Place Value

By reviewing place value strategies, educators help learners develop a strong foundation in understanding place value with decimals and enhance learner proficiency with all operations involving decimals. Reviewing place value with decimals can be incorporated into warms ups throughout the school year.

Learners should understand the relationship between ones, tenths, hundredths, and thousandths prior to being expected to complete operations with decimals. It is important for educators to understand the expectations at each grade level regarding number magnitude.

Instructional Strategies:

1. Use concrete models, such as base-10 blocks, to represent decimals. For example, represent tenths with rods and hundredths with unit blocks. This hands-on approach helps learners understand the relationship between the value of each digit and its position.
2. Utilize number lines to illustrate the placement of decimals.
3. Break down decimals into their expanded form. For example, the number 0.345, the 3 digit represents three tenths, the 4 digit represents four hundredths, and the 5 digit represents five thousandths.
4. Practice comparing and ordering decimals. Use models and representations such as base ten blocks and number lines to record answers. Encourage learners to justify why one decimal is greater or lesser than another based on their place value.
5. Connect decimals to real-life examples to make the concept more tangible. For example, relate decimals to money, measurements, or data analysis.
6. Provide regular practice opportunities for learners to reinforce their understanding of place value with decimals. Include a variety of problems that gradually increase in complexity to scaffold their learning effectively.

Equivalents: Fractions and Decimals

Learners are required to have prior knowledge with the concepts below to determine fraction and decimal equivalents:

- represent fractions and decimals using models, pictures, words, and symbolically
- generate equivalent forms of benchmark decimals and fractions
- compare decimals and fractions to common benchmarks
- understand magnitude by being able to compare and order decimals and fractions.

The connection between decimals and fractions is developed conceptually when learners' read decimals as fractions and represent them using visuals. For example, 0.8 is read as eight-tenths and can be represented using fraction strips or decimal strips (Wheatley and Abshire, 2002). Learners should use a variety of materials to model and interpret decimal tenths and hundredths.

Use t-charts and double number lines as pictorial ways to organize work. This practice of organizing helps learners to clearly articulate their thinking and recognize mathematical patterns. Misconceptions will lessen as learners are given opportunities to explore connections between models, oral, and written forms of decimals and fraction equivalents. It is beneficial to examine the connection made between decimals and base ten fractions to understand **decimal equivalence** (e.g., 0.3 = or $\frac{3}{10}$ or $\frac{30}{100}$ or $\frac{300}{1000}$).

Instructional Strategies:

1. Explore the relationship between fraction and decimal benchmarks. For example, 0.5 is another name for $\frac{1}{2}$; 0.25 is another name for $\frac{1}{4}$; 0.75 is another name for $\frac{3}{4}$.
2. Create sets of fraction and decimal cards. Partners could compare, sequence, and match card decks to practice strengthening their competency with this concept. Cards should begin with friendly numbers and benchmarks.
3. Represent decimals in a variety of ways. For example: 0.452 is $\frac{452}{1000}$ and can be expressed as 0.4 + 0.05 + 0.002 or $\frac{4}{10} + \frac{5}{100} + \frac{2}{1000}$.
4. Solve open-ended questions with fractions and decimals integrated into the question. For example: Your friend baked a pan of brownies. They offer to share the brownies with you, and below are the amounts they suggested. Which would you choose and why? Justify your thinking.

   a) $\frac{3}{5}$ b) 0.30 c) $\frac{5}{10}$ d) 0.7

Repeating and Terminating Decimals

A **repeating decimal** is a decimal representation of a **rational number** in which one or more digits repeat infinitely. The repeating part of the decimal is indicated by a **vinculum** (a horizontal line) placed over the digits that repeat, or sometimes by enclosing the repeating digits in parentheses. For example, in the representation 0.333..., the digit 3 repeats infinitely; it can be written as $0.\overline{3}$ with a vinculum over the 3 to indicate the repetition. Repeating decimals can occur in various patterns, such as single-digit repetition (e.g., 0.333...), multiple-digit repetition (e.g., 0.142857142857...), or mixed patterns (e.g., 0.121212...).

A **terminating decimal** is a decimal number that ends or stops after a certain number of digits past the decimal point. For example, the fraction $\frac{1}{4}$ has a decimal equivalent of 0.25. In this example, the decimal stops after two digits, so it is a terminating decimal.

Instructional Strategies:

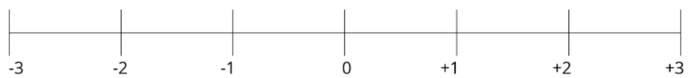
1. Use representative models, such as grids or charts, to represent fractions as decimals. Have learners shade in grids to represent fractions and then convert them to decimals.
2. Practice partitioning when dividing. Remind learners that fractions are represented as part-whole, part-part, measurement, operators and as **division**. Encourage learners to notice patterns in the quotients and remainders. If the division terminates (remainder becomes zero), then the decimal is terminating. If the division produces a repeating pattern in the quotient, then the decimal is repeating.
3. Recognize patterns in decimals. Show learners examples of both terminating and repeating decimals and ask them to identify the patterns. Encourage them to look for recurring groups of digits in repeating decimals.
4. Use examples that are in a realistic context, such as measurements, money, or calculations involving ratios. Discuss whether these decimals terminate or repeat, and why.

Integers: An Introduction The Math Improvement Site hosts quick reference guides to support this concept.
In middle school integers as a concept is introduced only to be further developed in high school mathematics. Integers are an extension of the whole number system, encompassing positive whole numbers, negative whole numbers, and zero. Investigating the concept of integers with linear models is recommended. By using a linear model, learners develop an understanding of the mirror image of integers and explore the balance and symmetry in math.

Learners will be able to imagine a number line stretching out infinitely in both directions, with zero in the middle. Then they will be able to discuss the mirror image of integers on any number line.

- Positive Integers (+): These are the numbers to the right of zero (e.g., +1, +2, +3, and so on).
- Zero (0): This is the midpoint on the number line.
- Negative Integers (-): These are the numbers to the left of zero (e.g., -1, -2, -3, and so on).

Using a linear model (horizontal and vertical), learners will see a symmetrical pattern. For every positive number on one side of zero, there's a corresponding negative number on the other side. It is like a reflection in a mirror where everything on one side has a matching partner on the other side.



Key understandings about integers include:

- zero is neither positive nor negative
- read comparisons like a sentence (e.g., “7 < 8”, reads like *seven is less than eight*; “0.54 > 0.16”, reads like *fifty-four hundredths is greater than sixteen hundredths*)
- negative integers are read as "negative 5" instead of "minus 5"
- positive integers may not always display the "+" symbol; if absent, the integer is positive
- negative integers mirror positive integers (e.g., +6 and -6 are equal distances from zero)
- negative integers are numerically less than positive integers
- closer positive integers to zero are less (e.g., +3 < +7)
- closer negative integers to zero are greater (e.g., -3 > -7)

Learners grasp the concept of integers better when taught in context, connecting to scenarios like height above or below sea level, temperature variations, banking transactions, etc. Contexts where negative integers are particularly useful include:

- temperature measurements, where below-zero temperatures are common
- golf scores, where scores can be above or below par
- financial situations involving debits and credits
- measurements of distance above and below sea level

Instructional Strategies:

1. Use open number lines in various orientations, horizontal and vertical, to help learners visualize the placement of integers.
2. Engage learners by having them categorize situations into Negative, Positive, or Zero contexts on a sheet of paper as they encounter them during lessons.
3. Create a human number line by distributing cards with positive and negative integers to learners and having them organize themselves in the correct order. Place an object on the floor to represent the zero. Learners then organize themselves in order compared to the zero.
4. Use the model of a thermometer, representing a vertical number line, to compare integers and record the comparisons symbolically. For example, -8 < 5; 6 > -7; 4 < 9; -3 > -4.
5. Explore in-context examples where negative integers are applied.

Percentage

Percentages represent ratios or fractions where the denominator is 100 (i.e., reflecting hundredths). The term "percent" indicates "out of 100" or "per 100". Percentages themselves do not denote a specific quantity; for instance, 90% can represent various fractions such as: $\frac{9}{10}, \frac{18}{20}, \frac{45}{50}, or\ \frac{90}{100}$.

Learners first explore percentages that range between 1% and 100%; then in **Grade 8** they explore percentages greater than 100%, and finally percentages less than 1% (fractional percents). They should be able to transition between percent, fraction, and decimal equivalents during problem-solving. For example:

- To find 25% of a number, simply take either $\frac{1}{4}$ of the number ***or*** divide the number by 4.
- 26% is equivalent to 0.26 (twenty-six hundredths) or $\frac{26}{100}$
- $\frac{3}{2}$ is equivalent to 150% ***or*** 1.5 (one and five tenths or fifteen tenths).

Prior knowledge of percentage benchmarks, and their fraction and decimal equivalents, make estimating and problem solving with easier.

Common Benchmarks and Their Equivalents		
Benchmark Percentages	**Decimal Equivalent**	**Fraction Equivalent**
100% (one hundred percent)	One Whole- recognize that 100% is a whole	the numerator and denominator are equal
75% (seventy five percent)	0.75 (seventy-five hundredths)	$\frac{3}{4}$ (three quarters)
50% (fifty percent)	0.5 (five tenths or fifty hundredths)	$\frac{1}{2}$ (one half)
33% (thirty-three percent)	0.333… (thirty-three hundredths)	$\frac{1}{3}$ (one third)
25% (twenty-five percent)	0.25 (twenty-five hundredths)	$\frac{1}{4}$ (one quarter)
20% (twenty percent)	0.2 (two tenths or twenty hundredths)	$\frac{1}{5}$ (one fifth)
12.5% (twelve and a half percent)	0.125 (one hundred twenty-five thousandths)	$\frac{1}{8}$ (one eighth)
10% (ten percent)	0.1 (one tenth)	$\frac{1}{10}$ (one tenth)
5% (five percent)	0.05 (five hundredths)	$\frac{5}{100}$ (five hundredths)
1% (one percent)	0.01 (one hundredth)	$\frac{1}{100}$ (one hundredth)
0.5% (one half of one percent)	0.005 (five thousandths)	$\frac{5}{1000}$ (five thousandths)

Remember that 50%, 5%, and 0.5% are NOT the same value.

Discussions should explore contexts where 1% could be considered significant and where 90% might appear minimal, emphasizing that these perceptions vary *depending on the magnitude of the whole.* For example, 1% of all the population of New Brunswick is a lot of people compared to 90% of the students in your class.

Learners will build upon percentage conceptual understanding by eventually solving problems involving finding a, b, or c in equations like a% of b = c, using estimation and calculation.

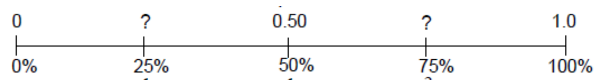
Problem solving scenarios could include:

- applying percentages to increase and/or decrease (e.g., taxes, discounts, probability etc.)
- applying percentages greater than 100 (e.g., interest rates, population growth etc.)
- applying understanding of percents to determine a number when a percent of it is known (e.g., sales commission, tip calculations, discounts etc.)
- finding the percent of a percent (e.g., tax on a sale price, discounts on discounts etc.)
- understanding when and how to use fractional percentages (percents less than 1%- grade 8)

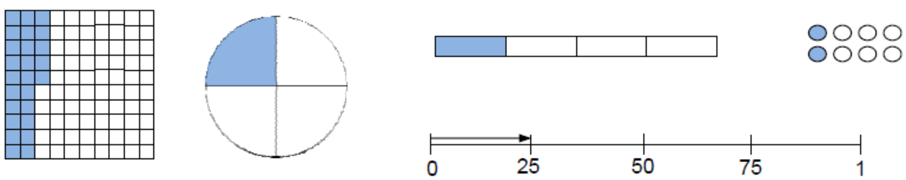
Instructional Strategies:

1. Use representations such as pie charts, bar graphs, and diagrams to illustrate percentages visually. This allows learners to see the relationship between the whole and its parts, making the concept more tangible and easier to comprehend.

- diagrams or grids, showing parts of a set, whole, or measurements over 100%
- 10 × 10 grid so learners have a visual of 1% - 100%
- double number lines to compare equivalents and/or to solve problems



- charts and tables that compare symbolic representations of equivalent fractions, decimals, and percentages



2. Explore the relationship between percentages and their equivalent decimals, equivalent fractions, and ratios (e.g., 48%, $\frac{48}{100}$, 0.48, 48:100).
3. Explore percent equivalents for common benchmark numbers and ratios.
4. Incorporate examples that are relatable to demonstrate the concept of percentages in context. For instance, show how percentages are used in sales discounts, taxes, and tip calculations. This reinforces the relevance and practical applications of percentages.
5. Incorporate hands-on models that engage learners in exploring percentages. For example, use models such as fraction bars to represent percentages visually.
6. Teach learners to use benchmarks to estimate percentages quickly and accurately. Provide practice exercises that require learners to estimate percentages in various contexts, reinforcing their understanding of relative sizes. A double number line is the ideal representative model for comparing percentages with fractions or decimals.
7. Emphasize the connections between percentages, fractions, and decimals. Practice converting between these representations and use visuals like t-charts or tables to organize learner thinking.
8. Calculate the percent of a number by:
   - convert a percentage to its' decimal equivalent and multiply
     (e.g., 110% of 80 = 1.1 × 80 = 88 **OR** 10% of 80 = 8 + 80 = 88)
   - use benchmarks to find a percentage of a number, for example:
     0.5% of 800= □
     Find 1% (one percent) of 800 and halve the product.
     0.5% of 800= □
     1% x 800 = 8
     8 ÷ 2 = 4
     0.5% x 800 = 4

     Understanding the relationship between 1% and 0.5% here makes the calculation easier.
   - change a percentage to a unit fraction, then divide:
     25% of 60

     Understanding that 25% = $\frac{1}{4}$, and that to find $\frac{1}{4}$ of a number you divide that number by 4.

$60 \div 4 = 15$
$25\% \; of \; 60 = 15$

- partition percentages (e.g., 15% could be thought of as 10% + 5%)
- think proportionally (e.g., 12% of 80 → $\frac{12}{100} = \frac{x}{80}$).

Ratios and Rate

Learners develop proportional reasoning by understanding and comparing multiplicative relationships between quantities, often depicted as ratios. This understanding progresses as learners connect ratios with fractions and percentages.

Ratios are comparisons between at least two quantities and can represent **part-to-whole** or **part-to-part** relationships. Ratios often expressed symbolically and can be used in various contexts, such as map scales. A ratio like "two to five" signifies that for every 2 items, there are 5 items.

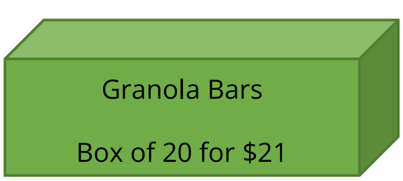
For example: The heart and faces below can be compared in many ways.
Part-to-Part Ratio: faces to hearts is 4:1
Part-to-Part Ratio: hearts to faces is 1:4
Part-to-Whole Ratio: faces to all is 4:5
Part-to-Whole Ratio: all to hearts is 5:1

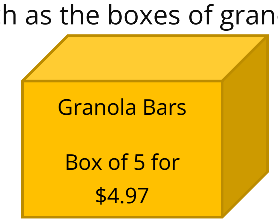
Understanding that **only part-to-whole ratios can be represented as fractions** is important because the denominator always refers to the whole. **A part to whole ratio can also be expressed as a percent** because ratios compare a part to a whole. For example, a problem involving teenagers attending a dance, the ratio compares the part (teenagers attending) to the whole (the entire school population), allowing it to be expressed as a percent.

Exploring equivalent ratios parallels exploring equivalent fractions, and learners can enhance their understanding by generating and solving their own problems.

Types of Ratio Scenarios	
Geometric	• The ratio of sides in a hexagon to a square is 6:4. • The ratio of vertices to edges in a rectangular prism is 8:12. • The ratio of vertices to sides in a hexagon is 6:6.
Numerical	• The ratio of a quarter's value to a dime's value is 25:10. • The ratio of *multiples of 2* to *multiples of 4* from 1 to 100. (2:1 or 50:25)
Measurement	• The ratio of perimeter to side length in a square is 4:1. • The ratio describing scale models or map scales is 1:15.

A ratio comparing measures of two different types (e.g., distance and time) is called a **rate.** Finding a **unit rate** is a common and useful application of this concept. For example: calculating the unit rate is used when comparing prices to find a better buy; such as the boxes of granola bars shown below:





Rates cannot be expressed as a percent because rates compare two different entities (e.g., such as speed which is in kilometers per hour).

Proportions are statements of equality between two ratios, and different notations can be used to express them. Solving a proportion involves finding one number when the other three are known.

For example, *proportions with ratios*: if there are 12 boys in a class with a boy-to-girl ratio of 3:5, you would set up the proportion $\frac{3}{5} = \frac{12}{x}$ and solve for *x*. This requires thinking multiplicatively, like determining equivalent fractions.

For example, *proportions with rate*: Proportional thinking is evident when a learner understands that a runner with a rate of 1 km/7 min will outperform a runner with a rate of 1 km/8 min in a 10 km race.

Instructional Strategies:

1. Use models, such as two-sided counters, coloured tiles, pattern blocks, linking chains etc., to introduce the concept of ratio as a comparison between two numbers. For example, in a group of 3 cats and 2 dogs:
    - 3:2 tells the ratio of cats to dogs (part-to-part)
    - 3:5 tells the ratio of cats to the total group (part-to-whole)
    - 2:5 tells the ratio of dogs to the total group (part-to-whole)
    - 2:3 tells the ratio of dogs to cats (part-to-part)
2. Practice proper math language: learners should read "3:2" as "3 to 2" or "3 __ for every 2 ___."
3. Use ratio tables to model proportion.
4. Explore ratios that occur in daily situations (e.g., water to concentrate when making juice is 3:1 or "3 to 1").
5. Present a collection of items that all have something in common (e.g., types of sports balls). Ask learners to describe the comparisons as ratios using part-to-part and part-to-whole ratio notation. *(Note: part-to-part ratios cannot be written as fractions).*

Percent of a Percent

This type of problem, introduced in **grade 8,** is common in everyday life. The prior knowledge required to solve percent-of-a-percent problems includes foundational fact knowledge, operations with decimals, understanding decimal place value, percentages, conversion between percentages and decimals, equivalence between fractions and decimals, and benchmarks for all number systems.

An example of a "percent of a percent" problem could be a salesperson earning commission. Suppose there is a sales team, and each member receives a commission based on their sales performance. Their commission rate is 5% of their total sales, and then the company offers them a bonus based on their commission earned. The bonus is 10% of the commission they earned.

Simple Example: What is 20% of 30%?

- Convert 30% to a decimal: 30% = 0.30.
- Find 20% of 0.30: 20% of 0.30 = 0.20 × 0.30 = 0.06.
- Convert 0.06 back to a percentage: 0.06 = 6%.
- Answer: 20% of 30% is 6%.

Commission Example: Calculate the bonus for a salesperson who earned $10 000 in commissions.

- Calculate the commission earned: $10 000 x 0.05 = $500 (5% of $10 000)
- Calculate the bonus based on the commission earned: $500 x 0.10 = $50 (10% of $500)
- So, the bonus for this salesperson would be $50.

Retail Discount Example: A store offers 10% off on a product that is already discounted by 25%. What is the total discount?

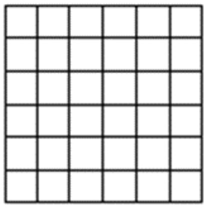
- Original discount: 25% = **0.25.**
- Additional discount on the already discounted price: 10% of 0.25 = 0.10 × 0.25 = **0.025.**
- Add the discounts together: 0.25 + 0.025 = 0.275.
- Convert to a percentage: 0.275 = 27.5%.
- Answer: The total discount is 27.5%.

Instructional Strategies:

1. Step-by-Step Approach: Break the problem into smaller, manageable steps.
    - Step 1: Convert the first percentage to a decimal.
    - Step 2: Multiply the second percentage by the decimal from Step 1.
    - Step 3: Convert the result back to a percentage, if necessary.
2. Use concrete and pictorial models (e.g., fraction bars, diagrams, charts etc.) to illustrate how percentages are calculated.

Perfect Squares The Math Improvement Site hosts quick reference guides to support this concept.

In **grade 8**, learners gain an understanding of perfect squares and square roots through various visual models like colour tiles or grid paper. Connecting concrete representations to numerical concepts results in concept mastery. *Emphasis on viewing the area of a square as a perfect square and any side length as its square root, fosters a deeper understanding of this mathematical concept.* In the figure below, learners are encouraged to view the area as the perfect square, and any dimension of the square as the square root.



Not all whole numbers are perfect squares and that the gap between perfect squares increases as the numbers being squared get larger. Learners should recognize that the differences between perfect squares increases in a consistent way as shown in the pattern below:

In addition to recognizing existing patterns, learners are encouraged to explore other perfect squares. Using **prime factorization,** prior knowledge of **prime factors,** and experience using **factor trees** will deepen understanding of this concept and strengthen problem-solving skills.

For example, $\sqrt{144}$.
Since $144 = 2 \times 72$
$= 2 \times 2 \times 36$
$= 2 \times 2 \times 6 \times 6$ [the process could be stopped at this point if students recognize this as 12×12: $(2 \times 6) \times (2 \times 6)$]
$= 2 \times 2 \times 2 \times 3 \times 2 \times 3$
$= (2 \times 2 \times 3) \times (2 \times 2 \times 3)$ [group factors in two equal groups]
$= 12 \times 12$, then $\sqrt{144} = 12$.

Learners must understand that numbers often have square roots that are decimal approximations, falling between two whole number square roots. *It's crucial to distinguish between an exact square root and a decimal approximation.*

For any **non-perfect square**, its square root will be an **irrational number**, meaning it cannot be expressed as a fraction and results in a non-repeating, non-terminating decimal. Even if an irrational number is approximated to a certain number of decimal places, it remains an approximation, as exemplified by **π** being approximately equal to *3.1415*.

Instructional Strategies:

1. Explore the reciprocal connection between squares (such as 3^2) and square roots ($\sqrt{9}$).
2. Offer learners chances to model thinking with concrete and pictorial representations of perfect squares (e.g., number lines, tiles, grids, etc.).
3. Ensure that learner are familiar with the perfect square benchmarks from 1 to 144 as these are used to establish an initial estimate when finding a square root (i.e., 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144).
4. Investigate patterns associated with perfect squares, like how the sum of the square roots of two consecutive perfect squares equals the difference between those squares.
   For example: $\sqrt{36} + \sqrt{25} = 6 + 5 = 11$ and $36-25=11$
5. Apply pattern knowledge to determine that the square root of 1600 is 40.
   For example: Apply prior knowledge that the square root of 16 is 4.
   $\sqrt{1600} = \sqrt{16} \times \sqrt{100} = 4 \times 10 = 40$
6. Practice estimation to enhance an intuitive grasp of square roots.
7. Use benchmarks (roots of perfect squares) between 1 and 144, to determine which two whole numbers encompass the square root and which whole number it's closer to.
   - For example: The square root of 22 is between 4 and 5 because 22 is between 16 and 25.

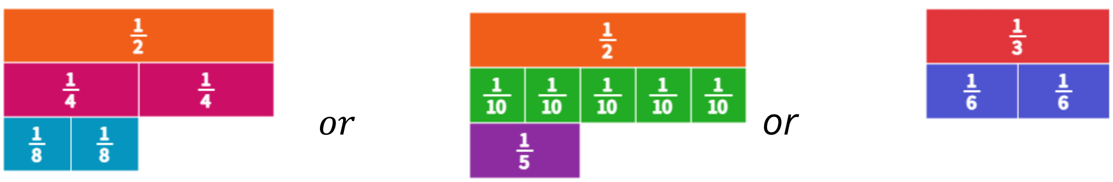
- For example: A whole number whose square root is between two given numbers such as 5 and 6. Any whole number between 25 and 36 possesses a square root between 5 and 6.

8. Use a calculator to estimate the square root of a non-perfect square without using the √ key. If learners are asked to estimate the square root of 110, they should know it is about halfway between 10 and 11, since 110 is almost halfway between 100 and 121.

Concept Elaboration: Operations

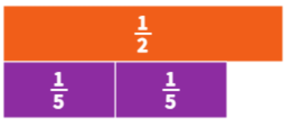
Fractions: Simplifying and Equivalents

Understanding the concepts of benchmark and unit fractions is an integral part to a learner's overall success when simplifying fractions. Fraction benchmarks or "friendly unit fractions" are:

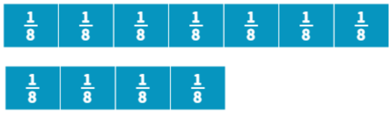


Learners should continue to use conceptual methods to compare fraction, including:

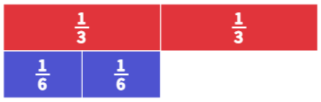
- comparing fractions to a benchmark (e.g., is $\frac{2}{5}$ more than or less than $\frac{1}{2}$);



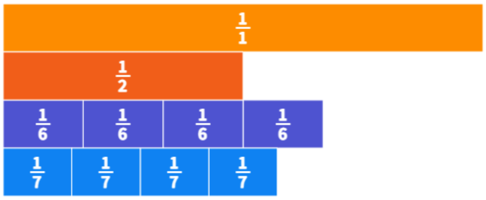
- comparing two numerators with the same denominator (e.g., $\frac{4}{8}$ and $\frac{7}{8}$);



- comparing two denominators when the fractions have the same numerator (e.g., $\frac{2}{3}$ and $\frac{2}{6}$).



A common error made by learners is to think because of their experience comparing whole numbers that a larger denominator means the fraction is larger (e.g., they think $\frac{4}{7}$ is greater than $\frac{4}{6}$).



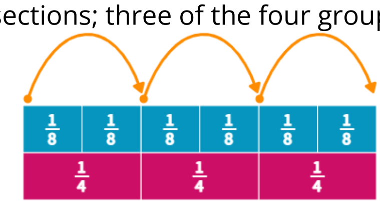
Learners should recognize that a fraction can **name part of a set** as well as **part of a whole** and the size of these can change. Learners need to understand that fractions can only be compared if they are parts of the same whole. For example: half of a cake cannot be compared to half of a cookie.

It is important that learners **visualize equivalent fractions** as the naming of the same **region** or **set** partitioned in different ways. Learners should be given opportunities to explore and develop their own strategies for creating equivalent fractions. They should be able to explain their strategy to others.

Rules for multiplying numerators and denominators to form equivalent fractions should not be provided to learners to follow without a conceptual understanding of why these rules work. A solid understanding of fractions is necessary before focusing on procedural rules for determining common denominators.

<u>Instructional Strategies:</u>

1. Provide learners with a variety of activities that include different interpretations of fractions: 1) part of a whole (e.g., part a chocolate bar); 2) a measure (e.g., part of a 4 m piece of rope).
2. Provide opportunities to model fractions both concretely and pictorially, using a variety of models such as, pattern blocks, grid paper, fraction pieces, fraction towers, counters, Cuisenaire rods, number lines, etc.
3. Point out to that to rename $\frac{6}{8}$ as $\frac{3}{4}$, you can "group" the 8 sections of the whole into twos. There are then four groups of 2 sections; three of the four groups are shaded.

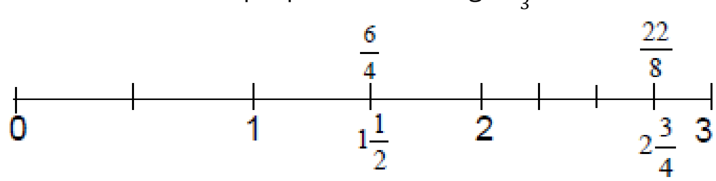


Equals three fourths ($\frac{3}{4}$)

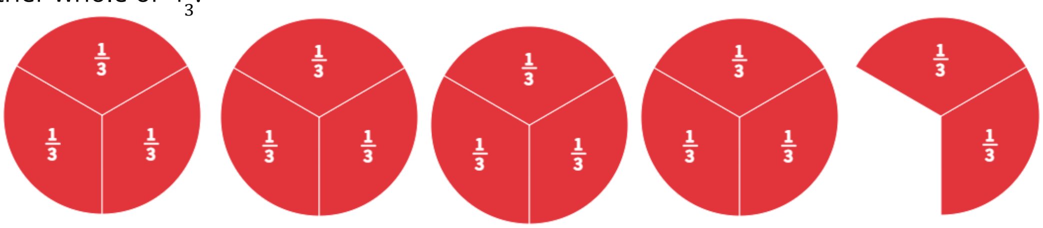
4. Use number lines and other models to compare fractions and explore equivalencies.

<u>Improper Fractions and Mixed Numbers</u>

In middle school, learners expand their grasp of fractions to recognize that an **improper fraction** (e.g., $\frac{8}{2}$) signifies a value exceeding one. By using models, learners can discern that fractions where the numerator exceeds the denominator are indeed greater than one. It is important for learners to understand that an improper fraction can be alternatively represented as a **mixed number**, comprising both a whole number and a proper fraction (e.g., $2\frac{2}{3}$).



Rather than relying solely on applying a conversion rule, learners should be urged to concentrate on the underlying meaning. For instance, considering that $\frac{14}{3}$ equals 14 thirds, and it requires 3 thirds to form 1 whole, 12 thirds would amount to 4 wholes. Thus, $\frac{14}{3}$ signifies 4 wholes and an additional 2 thirds of another whole or $4\frac{2}{3}$.

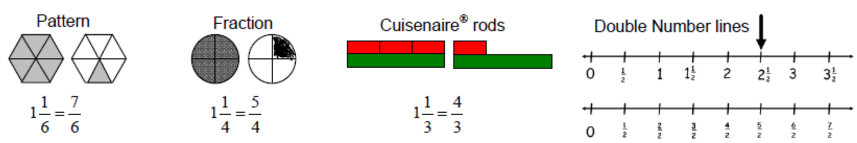


Learners should develop the ability to position mixed numbers and improper fractions on a number line, aided by benchmarks such as proximity to zero, nearness to one-half, closeness to one, and so forth. These benchmarks assist in visually understanding the arrangement and sequence of these fractions.

Through exploration, learners realize that dividing the numerator by the denominator can transform an improper fraction into a mixed number. Simply instructing learners to divide the denominator into the numerator without first fostering conceptual understanding would require them to properly recall the procedure without knowing why it works.

Instructional Strategies:

1. Explore improper fractions and mixed numbers in a variety of ways and use a variety of different models. For example:



2. Use pattern blocks and build and count fractional parts and continue beyond a whole. Ask learners to show another way to represent the improper fractions. Gradually transition to doing this activity without the pattern blocks (or other models).
3. Provide learners with frequent opportunities to use number lines (including double number lines) to explore the placement of mixed numbers and improper fractions. Ensure learners can justify their strategy focusing on the use of benchmarks.

Fractions: Addition and Subtraction The Professional Learning Hub hosts professional learning for educators including modules on fractions.

The concepts underlying operations with fractions mirror those of whole numbers, concepts supporting fractions and operations with fractions are introduced to learners in grades 4 and 5.

Almost all aspects of estimating fraction calculations rely heavily on understanding both the operations and fractions themselves. Incorporating estimation into computational learning is vital to ensure learners remain focused on comprehending the operations' meanings and the anticipated magnitude of the outcomes. Understanding and using unit fractions (e.g., numerator is always 1 and the denominator is always a number greater than 1) and benchmark fractions (e.g., $\frac{1}{4}, \frac{1}{2}, \frac{1}{10}$, etc.) are of key importance when estimating sums and differences of fractions.

After learners have investigated adding and subtracting fractions using concrete models and linear representations, they should transition to representing these calculations symbolically and delve into algorithms. When the denominators are uncommon, learners must utilize their previous understanding of factors to find common denominators and simplify fractions and mixed numbers.

<u>Instructional Strategies:</u>

1. Explore sums and differences of fractions by using a variety of concrete and linear models.
2. Connect problems applying the addition and subtraction of fractions and mixed numbers to similar problems with whole numbers. Include various structures of problems for addition and subtraction such as, part-part-whole and comparison from previous grades such as: $\frac{1}{2}+\square=\frac{5}{8}$ or $\square+\frac{1}{4}=\frac{6}{8}$.
3. Estimate sums and differences of fractions before calculating using benchmarks (e.g., $\frac{1}{4}, \frac{1}{2}, \frac{1}{10}$,1, etc.).
4. Focus on friendly fraction clusters in grade six prior before introducing problems that require multi-step computation. Learners must be competent and consistent with friendly fractions and their equivalent before learning an algorithm.
5. Ensure that learners record all solutions to fraction computations in the simplest form.

<u>Fractions: Multiplication and Division</u> The [Professional Learning Hub]() hosts professional learning for educators including modules on fractions.

Consider the following principles when developing personal strategies for multiplication and division of fractions:

- avoid hastily using computational rules before conceptual understanding is developed
- start with straightforward contextual tasks (such as sets, area models, and distance)
- establish connects between whole number and fraction calculations
- incorporate estimation and informal techniques
- investigate each operation using models

Incorporating estimation into computational learning is essential to maintain learners' focus on understanding the operations' meanings and the anticipated size of solutions. Connecting the multiplication of fractions to authentic problem-solving situations will enhances learner comprehension.

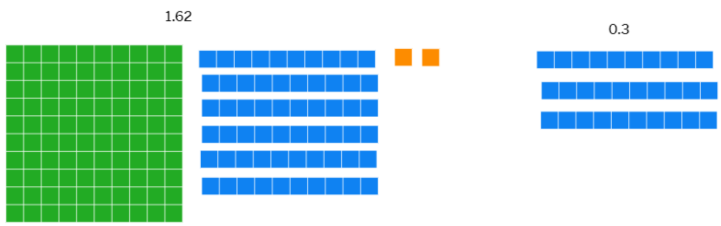
Instructional Strategies:

1. Discuss five ways in which fraction are most used: measurement, part to part, part to whole, quotient, and operator.
2. Connect the meaning of fraction computation to whole number computation.
3. Explore each of the operations using models (e.g., number lines, area model)
4. Use number lines to model what occurs when a fraction is multiplied by a whole number.
5. Use an area model for modeling multiplying mixed fractions.
6. Begin with common benchmark fractions when introducing fraction multiplication.
7. Present division of a fraction by a whole number as a sharing situation.
8. Estimate the quotient of positive fractions by using whole number benchmarks. (e.g., $7\frac{9}{10} \div 2$ is approximately $8 \div 2$, so the estimated quotient is 4).
9. Ensure that learners can compare the solutions of problems such as $8 \div \frac{1}{2}$ and $8 \times \frac{1}{2}$.
10. Review of order of operations rules for whole numbers with learners.

Decimals: Addition and Subtraction The Math Improvement Site hosts quick reference guides to support this concept.

Before operations with decimals learners must understand that all the properties and techniques used for operations with whole numbers. Learners should realize that adding or subtracting tenths (like 3 tenths plus 4 tenths equals 7 tenths) mirrors adding or subtracting quantities of other items (such as 3 apples plus 4 apples equals 7 apples). This concept extends to hundredths as well.

Instead of simply instructing learners to align decimals vertically or suggesting they "add zeroes," they are encouraged to consider the significance of each digit and how the parts combine. For instance, in the expression 1.62 + 0.3, a student might think of it as 1 whole, 9 tenths (6 plus 3), and 2 hundredths, resulting in 1.92.



To effectively estimate sums and differences, learners must have regular exposure to different scenarios to ensure learners have ample practice. When faced with a problem requiring an exact answer, learners should assess if they can use friendly numbers to calculate it beforehand; this assessment should occur each time a calculation is necessary.

- **Compatible number strategy**, when the learner relies on prior whole number knowledge of compatible (partner) numbers to determine the sum or difference. (e.g., 0.72 + 0.23 are close to 0.75 and 0.25)
- **Front-end addition or subtract strategy** when learners connect their understanding of place value and magnitude to determine a sum or difference.

- e.g., 32.3 + 24.5 + 14.1= ?
- "30 + 20 + 10 = 60"
- " 23 tenths + 45 tenths + 41 tenths = **109 tenths or 10.9**" **understanding of decimal equivalences**
- "60 + 10.9 = 70.9"

- **Constant difference strategy** states that the same quantity can be added to (or subtracted from) each number in a computation without affecting the answer (difference).
- **Counting on or counting back** as a strategy can be supported by a number line representation.

Instructional Strategies:

1. Provide opportunity for learners to model and solve addition and subtraction questions involving tenths, hundredths, and thousandths concretely, pictorially, and symbolically (e.g., thousandths and hundredths grids, base ten blocks, and number lines).
2. Present addition and subtraction questions both horizontally and vertically to encourage alternative computational strategies. For example, for 1.234 + 1.990, learners might calculate: 1.234 + 2 = 3.234 followed by 3.234 - 0.01 = 3.224.
3. Have learners investigate the relationship between adding decimals numbers and whole numbers. For example, 356 + 232 = 588; this looks like 0.356 + 0.232 = 0.588.
4. Provide problem solving situations that require students to add or subtract decimals using a variety of strategies.
5. Have learners determine an estimate when solving problems involving the addition and/or subtraction of decimals.

Decimals: Multiplication and Division

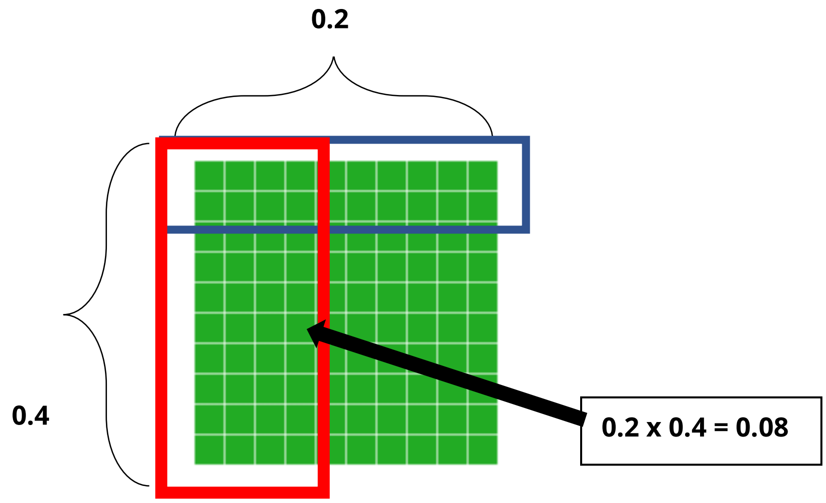
As learners explore multiplying and dividing with decimals the skill of estimation helps ensure answer reasonableness. When exploring decimal multiplication, learners should understand that, for instance, 0.9 of a quantity will be nearly the whole or 100% of that amount. Also, multiplying by 2.4 will result in double the quantity plus almost an additional half of that quantity. Emphasizing the usefulness of estimation is essential and regular practice is encouraged.

Front-End Estimation

- Multiplication: 6 × 23.4 might be 6 × 20 (120) plus 6 × 3 (18) plus a little more for an estimate of 140, or 6 × 25 = 150.
- Division: 424.53 ÷8 = ☐ Learners might estimate that 50 × 8 = 400, so the quotient of this equation must be a bit more than 50.

Area Model

Using the area model for multiplication and division with decimals involves breaking down each decimal digit by place value and representing them as areas within a rectangle or grid. Learners begin by using a grid of 10 x 10 for this type of operation representation. Decomposing the decimal by place value is required to use this method of representing multiplication and division. For example, multiplying 3.2 by 2.5, decompose 3.2 into 3 whole units and 0.2, and decompose 2.5 into 2 whole units and 0.5.



The area model provides a visual representation of multiplication and division with decimals, making the concepts more tangible and easier to understand for learners.

Establishing a relationship between multiplication and division is crucial, as multiplication can serve as a strategy when estimating quotients. For instance, consider 74.3 divided by 8. Encourage learners to identify the multiples of 8 closest to 74.3. Demonstrate the calculations: 8 × 9 = 72 and 8 × 10 = 80. Learners should articulate why the quotient lies between 9 and 10. It is important to use precise vocabulary when reading numbers, aiding learners in linking facts together (e.g., 4 × 6 is akin to 4 × 0.6; representing 4 groups of 6 tenths equals 24 tenths or 2.4).

Instructional Strategies:

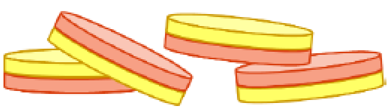
1. Use proper vocabulary related to multiplication (factors, product) and division (divisor, dividend, quotient).
2. Look for benchmark decimals that are easy to multiply and divide. For example, ask the learners why someone might estimate 516 × 0.48 by taking half of 500.
3. Provide opportunities to create and solve missing factor and missing divisor/dividend problems, involving decimals, to support the connection between multiplication and division.
4. Use the "area model" both concretely with base ten blocks and pictorially to represent multiplication and division before moving to the symbolic.

5. Use patterns to help learners understand the placement of the decimal in the product of two decimal amounts. For example, 9 x 7 = 63 therefore, 9 x 0.7 (or 7 tenths) = 6.3 or 63 tenths.
6. Focus on strategies such as rounding and front-end estimation.

Order of Operations The Math Improvement Site hosts quick reference guides to support this concept.

Integers: Addition and Subtraction The Math Improvement Site hosts quick reference guides to support this concept.

Integers encompass whole numbers and their opposites, suggesting they handle both quantity (magnitude) and opposites (direction). Quantity is indicated by the number of counters or the length of the arrows on the number line, while opposites are represented by different colors or directions. Concrete representations frequently used for modelling addition and subtraction with integers are two-coloured counters, algebra tiles, and number lines. Chick here to check out this interactive lesson on how to use algebra tiles when adding integers.



Understanding and utilizing the **zero principle** is essential when adding or subtracting with integers. This principle demonstrates why any subtraction number sentence can be rewritten as an equivalent addition number sentence. The zero principle also involves the balance of positive and negative values to yield zero. For example, (-1) + (+1) = 0, (-3) + (+3) = 0, and (-17) + (+17) = 0, demonstrating that the sum of opposite integers results in zero.

In everyday life there are many uses of integers, so problem solving plays a major role in developing understanding of integer operations. Learners should see a connection between integers and their world around them using problems using real-life contexts such as height above and below sea level, temperature, and banking (deposits and withdrawals), etc.

Instructional Strategies:

1. Model thinking and strategies with concrete, pictorial representations, and finally progress to symbolic representations. (e.g., algebra tiles, two-coloured counters, number lines)
2. Connect the subtraction of integers to the addition of integers to reinforce understanding.
3. Relate problems involving addition and subtraction of integers to similar problems with whole numbers.
4. Incorporate different problem structures like "part-part-whole" and "comparison" from previous grades. Examples, $-6 - x = 3$

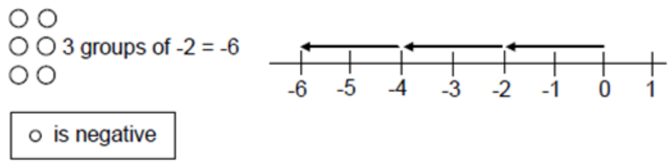
   $x + 2 = -5$

   $-9 = -4 + x$

5. Have learners justify strategies they use in finding sums and differences of integers and provide opportunities for discussing strategies used by their peers.

Integers: Multiplication and Division (2-digit by 1-digit) The Math Improvement Site hosts quick reference guides to support this concept.

Using models is necessary for understanding abstract concepts like multiplication and division with integers. Two useful models for illustrating these concepts are two-coloured counters and number lines. These models aid in visualizing the operations and deepen understanding of integer multiplication. For example, 3 × (-2) = -6 can be represented as shown below.



Patterning can also be used to justify the result for a negative multiplied by a negative.

3 × (-2) = -6
2 × (-2) = -4
1 × (-2) = -2
0 × (-2) = 0
-1 × (-2) = 2
-2 × (-2) = ?
-3 × (-2) = ?

Understanding the **commutative property** when multiplying integers, allows leaners to rearrange the order of multiplication without affecting the product. For example, (-4) × 5 can be interpreted as 5 groups of (-4).

Comparing multiplication and division situations is beneficial for understanding the division of integers. Once multiplication is fully understood, learners recognize that multiplication and division are inverse operations. For example, if -4 × 3 = -12, then it follows that the product divided by either factor should equal the other factor; thus, -12 ÷ (-4) = 3 and -12 ÷ 3 = -4. Similarly, if -4 × (-3) = 12, then 12 ÷ (-4) = -3 and 12 ÷ (-3) = -4. Using a missing factor can also aid in understanding these operations. **After** exploring multiplication and division with models, learners should be prompted to develop rules for determining the signs of products and quotients.

Division Statement		Related Multiplication Statement
-16 ÷ (-4) =	⇨	? × (-4) = -16

Finally, determining the rule for the sign of a product or quotient of integers, particularly through patterning, can be approached in various instructional strategies. Begin with simple integer multiplication problems, both positive and negative, and encourage learners to observe patterns in the signs of the products. For example, have learners multiply pairs of positive integers, pairs of negative integers, and then mixtures of positive and negative integers. Ask them to note any patterns they observe in the signs of the products.

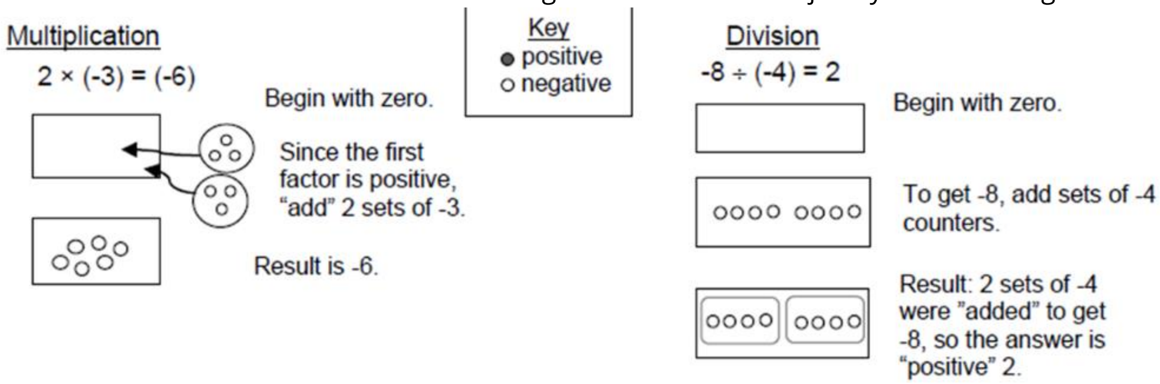
Guide learners to recognize that the *product of two positive numbers is positive*, the *product of two negative numbers is positive*, and the *product of one positive and one negative number is negative.* Encourage learners to create a visual representation of the patterns they observe.

Reflective Questions:

- Why does multiplying two positive numbers result in a positive product?
- What happens when we multiply a positive number by a negative number?
- How does division relate to multiplication in terms of the sign of the quotient?

Instructional Strategies:

1. Use models such as counters, number lines to demonstrate strategies.
2. Explore the concept of net worth.
3. Explore the importance of patterning. Patterning stands out as particularly effective for illustrating the multiplication or division of two negative numbers in a clear and accessible manner for students.
4. Have learners estimate products and quotients to check their reasonableness.
5. Use relevant contexts for multiplication and division of integers. For example, the impact on net worth if a person owes $6 to each of 3 friends **OR** if a debt of $6 to each of 3 friends is forgiven.
6. Have learners write the number sentence using concrete models to justify their thinking.



Cross-Curricular Connections

From the analysis of data collection, coding of computer systems, creation of musical composition, innovation in environmental science, or precision required in carpentry classes, mathematics core concepts enhance the proficiency of all learners in other subjects. Below are some examples of how teaching teams could connect mathematics with other curricular areas.

Grades	Mathematics Curriculum	Visual Art Skill Descriptor
6, 7, 8	Exploring the achievement indicators within the Big Idea of *2-D Shapes and 3-D Objects.*	Apply the elements of art and the principles of design to develop skills, language, techniques, and processes.
Grades	**Mathematics Curriculum**	**Science Skill Descriptors**
6, 7, 8	Exploring the achievement indicators within the Statistics and Probability strand.	Collect and represent data using tools and methods appropriate for investigations of natural and technical sensory systems. Analyze and interpret qualitative and quantitative data to construct explanations and conclusions. Collect and represent data using tools and methods appropriate for investigations of motion and stability, the laws of motion, and space exploration.

Assessment

Formative assessment can and should happen every day as an integral part of the teaching and learning process. Robust, mathematical assessment practices require educators to **communicate clear learning goals, provide varied opportunities for learners to demonstrate their understanding, analyze triangulated learner evidence consistently, provide specific and descriptive feedback to learners and reflect/adjust teaching strategies based on data.** Formative assessment is focused on the learning process and progress rather than only on achievement which perpetuates the myth that mathematics is a subject of right and wrong answers.

In addition to ensure all are aware of the learning goals, it is important to determine what learners know and can do already prior to instruction as well as which concepts and skills will require additional support. Having this information is essential in planning instruction that supports all learners' needs.

Providing feedback to learners in mathematics highlights strengths, offers guidance for improvement, and addresses learner mathematical misconceptions. It is most effective when it is focused on a few areas to improve, and time is given for learners to use the information. Feedback can also be given by peers. **Open-ended questions** incorporated into an assessment plan are recommended as they promote metacognitive thinking in mathematics.

Learner evidence in mathematics is inclusive of **products**, **conversations**, and **observations**. Observational learner evidence is the measure of what a learner is capable of independently, without prompting by the educator in a direct or indirect manner. This information, like other sources needs to be collected, examined, and used to adapt teaching and learning. When the data is recorded, it is available to be used to monitor the progress of learners.

"Formative assessments help students think about mathematics as a growth subject where there is a path towards greater understanding. It shows children their own path forward and avoids the inherent fixed messages of test scores and grades, "(Boaler, 2015).

LINGUISTICALLY INCLUSIVE ASSESSMENT

Consider the demands of your assessments. Are they measuring the content area knowledge and/or skills, as intended, or are they measuring English proficiency and reading skills? (Hyunh and Skelton, 2023).

Equitable assessment of learners still learning the language of the content area considers the following steps:

Use of Universal Design for Learning (UDL) Principles	Use of Scaffolds for Performance Based Assessments	Use of Scaffolds for Tests, Exams, and Quizzes
Provide opportunities for students to demonstrate learning using multiple means of expression. For example: • Allow learners to use visual aids, models, and technology to solve problems and represent their understanding.	Provide templates. Break down into steps and directly teach how to complete each. Build in sequencing, instruction boxes, and sentence starters/frames.	Add the following supports: • Multiple choice • Matching • Sentence frames/sentence starters • Chunking • Images • Word banks • Simplified instructions

• Provide graphic organizers to help learners organize their thoughts and problem-solving processes.	Grade only for content knowledge and content-related language specifically taught.	

Please refer to the Assessing, Evaluating, Reporting Holistic Curriculum K-8 Companion Document for an in-depth review of summative assessment and reporting information.

Planning for All Learners

A strength-based approach to mathematics instruction requires educators to know the strengths of their learners and scaffold the next steps of learning. Below are some suggested guides to help educators create strength-based and differentiated mathematics classrooms:

Personalized Learning

Culturally Responsive Teaching

CULTURALLY AND LINGUISTICALLY INCLUSIVE ENVIRONMENT

Culturally and linguistically inclusive environments are environments in which the variety of cultures (understood as "ways of being, knowing and doing," or what we do and why) and language skills and levels from which learners are approaching their education and the world, are recognized, respected, and honoured (NBEECD, 2020).

There are many actions educators take when enacting culturally and linguistically inclusive environments, but three key ones that span across all content areas are:

1. Use of translanguaging
2. Co-creation of classroom norms
3. Environment as co-teacher

Translanguaging: Strategically plan opportunities for **translanguaging,** allowing languages to work together to support learning. This can be achieved by alternating the language used during the input, processing, and output phases of a learning cycle (Crisfield, 2017). See example below:

Input	Processing	Output

Complete math strategy discussion handout in home language.	Share learning with peers during group discussion in math class, in English.	Create notes on strategies shared by peers in home language. Create infographic or list main ideas in English.

Co-creation of classroom norms: Co-create class and group norms. Learners who are newcomers to New Brunswick may be operating with different sets of unwritten rules for what appropriate behaviour and learning systems should look like in a school setting. Co-creating norms makes expectations explicit and allows for diverse voices to be incorporated. Co-creating norms may also appeal to newcomers from more collectivist cultures where group harmony and contributing to a sense of community are valued.

Environment as co-teacher: Strategically use classroom displays and routines to support students with the language of *mathematics,* such as:

Multilingual Word Walls	Anchor Charts	Annotated Objectives
Interactive word walls that change based on the topic, unit, or theme being studied. Include key words, terms, phrases in English and the other languages spoken in the class.	Poster highlighting key ideas, processes, or concepts related to a particular skill or unit and supported by visuals	Objectives for each lesson are shared with students who work with teacher to annotate the academic vocabulary. E.g., $2x = 10$ $2x$ is the variable term = means equal 10 is the product

TECHNOLOGY

Technology usage in mathematics classrooms supports discourse, intervention, and instruction; technology is not a replacement for any of the forementioned. Incorporating technology into mathematics classrooms requires intentional planning, creating a respectful environment for sharing ideas and facilitating meaningful discussions.

The Microsoft Office 365 suite has several accessibility features to support all learners. All the listed tools are available to support all educators and learners.

Translation tools:

- Translate selected text: Select words and phrases for translation, and simply right-click to see your translation in the Translator pane.
- Translate entire documents: Create a translated copy of documents with links intact by clicking the Review tab, and the Translate button.

Accessible templates for Word, Excel, PowerPoint: to find templates tagged with "Accessible," head to File > New in any of the Office apps. Enter Accessible in the search box at the top on the screen and press Enter to view all the results. To refine your search further, use the category listings in the right-hand pane. You can select multiple categories. Accessibility tools for Microsoft 365 - Microsoft Support

Screen formatting for visual impairments or for ease of reading: turn on Read Mode, go to View > Read Mode. The app will remove all the other clutter from your screen and only display the text. Click on the View menu for further options.

Using online digital resources: The digital environment allows access to resources for educators which provide educational opportunities. Accessing websites for educational purposes requires judicious decisions by educators, including respecting privacy and copyright. Software for educational purposes, including webapps, require risk assessment by the prior to use. Check with your district technology coordinator for approved software. More information can be found on the Digital Learning SharePoint.

Glossary

Chunking	Dividing complex information into manageable chunks and/or dividing complex language into phases with words that would be grouped together by fluent English speakers/readers.
Images	Pictures, diagrams, illustrations.
MLL	Multilingual Language Learner: A learner with a primary or home language other than English who is learning English as an additional language.
Scaffold/Scaffolding	The process of breaking lessons into manageable units, with the educator providing decreasing levels of support as learners grasp new concepts and master new skills.
Sentence Frames/Sentence Starters	Provides the framework for a complete sentence with blanks where students will fill in their own words. Can be used to support oral and/or written expression.
Simplified Instructions	Instructions provided in short, simple sentences with basic vocabulary.
Translanguaging	Using two or more languages together to support learning.
Word Banks	A written list of key vocabulary students can choose from to support oral and written output. Can be combined with sentence frames/sentence starters.

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